

٥٠١٥/٤/١٤ (عيد الحب)  
له أسفّر الله العظائم

سكنه راحة

م/منار

\* Find modulus & argument of:

$$\boxed{1} \quad z = \frac{z+i}{3+4i}$$

$$|z| = \left| \frac{2+i}{3+4i} \right| = \frac{|2+i|}{|3+4i|} = \frac{\sqrt{2^2+1^2}}{\sqrt{3^2+4^2}} = \frac{1}{\sqrt{5}}$$

$$\arg(z) = \arg \left| \frac{2+i}{3+4i} \right| = \arg(2+i) - \arg(3+4i)$$

$$= \tan^{-1}\left(\frac{1}{2}\right) - \tan^{-1}\left(\frac{4}{3}\right) = -26.565^\circ$$

$$\boxed{2} \quad \frac{1+2i}{3-4i} + \frac{2-i}{5i}$$

$$|z| = \left| \frac{1+2i}{3-4i} \right| + \left| \frac{2-i}{5i} \right| = \frac{|1+2i|}{|3-4i|} + \frac{|2-i|}{|5i|}$$

$$= \frac{\sqrt{1+4}}{\sqrt{25}} + \frac{\sqrt{5}}{\sqrt{25}} = \frac{5}{5 \times 5} = \frac{2}{5}$$

هنا نقوم بتوضيح  
المقامات

$\boxed{1}$

sheet

1 Find real & imaginary part & Find Polar form

$$Z = x + iy = r \{ \cos(\theta) + i \sin(\theta) \}$$

مطلوب واصل

$$a) z = (1 + \sqrt{3}i)^6$$

$$r = |z| = |(1 + \sqrt{3}i)^6| = |1 + \sqrt{3}i|^6 = (\sqrt{1+3})^6 = \underline{64}$$

$$\theta = \arg(1 + \sqrt{3}i)^6 = 6 \arg(1 + \sqrt{3}i) = 6 \tan^{-1}(\sqrt{3}) = 2\pi$$

$$Z = 64 [\cos(2\pi) + i \sin(2\pi)]$$

$$= 64 + 0i$$

real  $\rightarrow 64$

imaginary  $\rightarrow 0$

$$b) \left( \frac{1+i}{1-i} \right)^4$$

نفس حل المثال السابق

$$r = |z| =$$

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2] show that

$$a) 1 + \cos \theta + \cos i \theta + \dots + \cos(n\theta) = \frac{1}{2} + \frac{\sin(n + \frac{1}{2})\theta}{2 \sin(\frac{\theta}{2})}$$

$$e^{i\theta} = \cos \theta + i \sin \theta$$

$$\operatorname{Re} [e^{i\theta}] = \cos \theta$$

$$\text{L.H.S} = 1 + \cos \theta + \cos i \theta + \dots + \cos(n\theta)$$

$$= \operatorname{Re} [1 + e^{i\theta} + e^{i2\theta} + \dots + e^{in\theta}]$$

المجموع  
الهندسي  $a + ar + ar^2 + \dots = a \frac{1 - r^{n+1}}{1 - r}$

$$a = 1, r = e^{i\theta}$$

$$= \operatorname{Re} \left[ \frac{1 - e^{i\theta(n+1)}}{1 - e^{i\theta}} \right] * \frac{-i \frac{\theta}{2}}{-i \frac{\theta}{2}}$$

بالضرب في  $e^{-i \frac{\theta}{2}}$

$$= \operatorname{Re} \left[ \frac{e^{-i \frac{\theta}{2}} - e^{i(n+1) \frac{\theta}{2}}}{e^{-i \frac{\theta}{2}} - e^{i \frac{\theta}{2}}} \right]$$

$$= \operatorname{Re} \left[ \frac{\cos(\frac{\theta}{2}) - i \sin(\frac{\theta}{2}) - \cos[(n+1)\frac{\theta}{2}] + i \sin[(n+1)\frac{\theta}{2}]}{\cos \frac{\theta}{2} - i \sin \frac{\theta}{2} - \cos(\frac{\theta}{2}) - i \sin(\frac{\theta}{2})} \right]$$

$$\boxed{P}$$

$$= \frac{-\sin\left(\frac{\theta}{2}\right) - \sin\left(n+1\right)\frac{\theta}{2}}{-2 \sin \frac{\theta}{2}}$$

$$= \frac{1}{2} + \frac{\sin\left(n+\frac{1}{2}\right)\theta}{2 \sin\left(\frac{\theta}{2}\right)}$$

$$\begin{aligned} \text{b) } & \frac{(\cos 2\theta - i \sin 2\theta)^7 (\cos 3\theta + i \sin 3\theta)^{-5}}{(\cos 4\theta + i \sin 4\theta)^{12} (\cos 5\theta - i \sin 5\theta)^{-6}} \\ &= \cos 107\theta - i \sin 107\theta \end{aligned}$$

$$\begin{aligned} \text{L.H.S} &= \frac{\left(e^{-i2\theta}\right)^7 \left(e^{i3\theta}\right)^{-5}}{\left(e^{i4\theta}\right)^{12} \left(e^{-i5\theta}\right)^{-6}} = \frac{e^{-i14\theta} \cdot e^{-i15\theta}}{e^{i48\theta} \cdot e^{i30\theta}} \end{aligned}$$

$$= e^{i(-14-15-48-30)} = \frac{-i107\theta}{e}$$

$$= \cos(107\theta) - i \sin(107\theta)$$

E

[4] show that

$$|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$$

sol

$$\text{L.H.S} = |z_1 + z_2|^2 + |z_1 - z_2|^2$$

$$= (z_1 + z_2)(\overline{z_1 + z_2}) + (z_1 - z_2)(\overline{z_1 - z_2})$$

$$= (z_1 + z_2)(\bar{z}_1 + \bar{z}_2) + (z_1 - z_2)(\bar{z}_1 - \bar{z}_2)$$

$$= z_1 \bar{z}_1 + z_1 \bar{z}_2 + z_2 \bar{z}_1 + z_2 \bar{z}_2 + z_1 \bar{z}_1 - z_1 \bar{z}_2 - z_2 \bar{z}_1 + z_2 \bar{z}_2$$

$$\text{L.H.S} = |z_1|^2 + |z_2|^2 + |z_1|^2 + |z_2|^2$$

$$= 2|z_1|^2 + 2|z_2|^2$$

□

5) use De Moivre theorem to obtain  $\cos 3\theta$

&  $\frac{\sin 3\theta}{\sin \theta}$  in terms of power of  $\cos \theta$ .

Sol

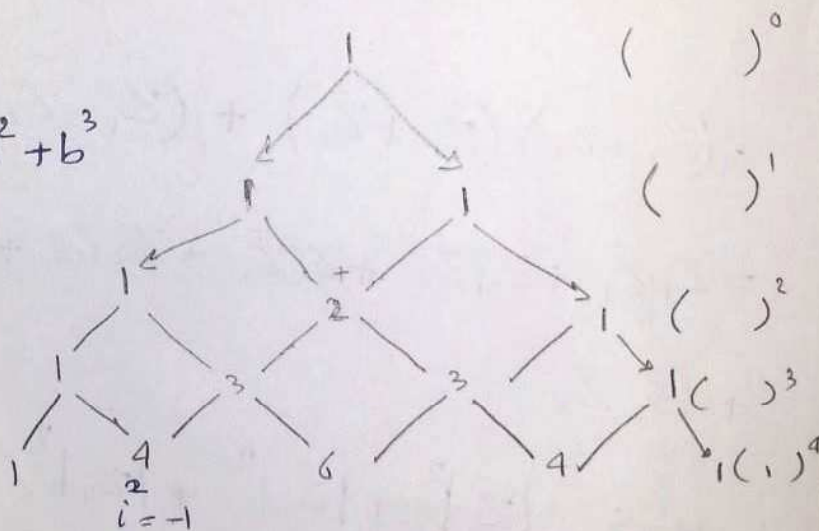
De Moivre  $\rightarrow (\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$

$$(\cos \theta + i \sin \theta)^3 = \cos 3\theta + i \sin 3\theta$$

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

$$a = \cos \theta$$

$$b = i \sin \theta$$



$$= \cos^3 \theta + 3i \cos^2 \theta \sin \theta - 3 \cos \theta \sin^2 \theta - i \sin^3 \theta$$

$$= \cos 3\theta + i \sin 3\theta$$

$$\cos 3\theta = \cos^3 \theta - 3 \cos \theta \sin^2 \theta$$

$$= \cos^3 \theta - 3 \cos \theta [1 - \cos^2 \theta]$$

$$= 4 \cos^3 \theta - 3 \cos \theta$$

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$$3 \cos^2 \theta \sin \theta - \sin^3 \theta = \sin 3\theta$$

$\sin \theta \neq 0$

$$\frac{\sin 3\theta}{\sin \theta} = 3 \cos^2 \theta - \sin^2 \theta$$

$$= 3 \cos^2 \theta - (1 - \cos^2 \theta)$$

$$= 4 \cos^2 \theta - 1$$

